2. Types of Cross Sectional Correlation

Baseline panel model:
\[ y_{it} = \rho y_{i,t-1} + u_{it}, \quad t = 1, \ldots, T, \quad i = 1, \ldots, N \]

Common Factor Models
- correlation through (unobserved), time-varying common factor
- \( u_{it} = \gamma_1 f_t + \epsilon_i \) or \( u_{it} = \Gamma f_t + \epsilon_i \) in vector notation
- \( \Gamma = [\gamma_1, \ldots, \gamma_N] \) are factor loadings
- \( f_t \) is (unobserved) common factor
- \( \epsilon_i \) is \( \sim \text{white noise error vector} \)

Contemporaneous correlation is then:
\[ \Omega_{y} = \Sigma_{y} = \text{Cov}(\epsilon_i, \epsilon_j) = \text{Cov}(f_t, f_j) + \text{Cov}(\epsilon_i, \epsilon_j) \]

\( \Omega_{y} \) characterizes strong dependence; largest eigenvalues of \( \Omega_{y} \) are of order \( O(N) \)

Spatial Error Models
- correlation decays with (economic) distance between cross section members
- \( u_{it} = B^\top \epsilon_i, \quad B = \text{lag distance} \quad \theta \rightarrow 0 \)
- \( \theta \) is spatial AR coefficient \( \Rightarrow |\theta| < 1 \)
- \( W \) is defined as spatial distances or weights
- spatial AR(p) model: direct split-overs to next \( p \) neighbors behind and ahead

3. Panel Unit Root Tests

All tests based on pooled regression:
\[ \Delta y_{it} = \gamma_1 y_{i, t-1} + \phi_i, \quad H_0: \phi = 0 \]

3.1 Existing Procedures

1st Generation Test (Levin, Lin & Chu, 2002)
- under assumed model without deterministic terms or serial correlated errors
- if \( \epsilon_i \) and \( \epsilon_j \) are free of contemporaneous correlation
- \( u_{it} \sim N(0,1) \) as \( T \rightarrow \infty, N \rightarrow \infty \)

2nd Generation Test (Breitung & Das, 2005)
- robustly LL test by explicitly estimating correlation pattern

\[ \Omega = \frac{1}{T} \sum_{t=1}^{T} u_i u_j' \quad \text{(N x N matrix)} \]

- under weak correlation

- \( \Omega \) singular if \( N \gg T \)
- in these instances \( \Omega \) bad approximation of \( \Omega \)
- potentially biased inference

3.2 Finite Sample Modifications

A 'White'-Type Test
- consider replacing \( \hat{\phi}_i \) in \( t_{SC} \) by panel 'White'-type covariance estimator

- the same asymptotic behavior as \( t_{SC} \), but
- more robust if \( N \gg T \)
- potentially robust against time varying covariances

Refined finite sample residuals
- MacKinnon & White (1985) and Davidson & Flachaire (2001): bias reduction of heteroscedasticity consistent covariance estimators in classical regression models through application of residuals

we adopt this approach and generalize it to panel case
- replace \( u_{it} \) in \( \hat{\phi}_i \) and \( \epsilon_i \) by \( u_{it}(\hat{\Omega}^{-1} - \hat{\Omega}^{-1} \hat{\Omega}^{-1}) \hat{u}_{it} \)

- call refined statistics \( t_{SC} \) and \( t_{SC} \)

3.3 The Wild Bootstrap

Basics
- wild bootstrap: construct asymptotically correct critical values (CVs) via re-sampling without replacement
- bootstrapped CVs might provide better approximations of true CVs than asymptotic approximations

consistency only relies on large \( T \)-asymptotics

The Algorithm
1. run pooled regression and obtain residuals
2. for \( s = 1, \ldots, S \) with \( S \) sufficiently large
3. draw bootstrap residuals
4. construct the bootstrap sample as \( y_{it}^* = y_{it}^* + \epsilon_{it}^* \)
5. calculate bootstrap version \( \hat{\phi}_i^* \) of original statistic \( \hat{\phi}_i \)
6. decision: reject \( H_0 \) at significance \( \alpha \)
7. different choices of distribution for \( \epsilon_i \)
8. following Davidson & Flachaire (2001), we chose Rademacher distribution:

\[ \eta_i = \begin{cases} \text{1 with Pr 0.5} & \text{1 with Pr 0.5} \\ -1 with Pr 0.5 & \text{1 with Pr 0.5} \end{cases} \quad \text{E}[\eta_i] = 0, \quad \text{E}[\eta_i^2] = 1, \forall i \]

5. Monte Carlo Study

Setup of Simulation
- data generated according to
- different patterns of error correlation
- DG1: \( u_{it} = c + \epsilon_{it} \text{ i.i.d N}(0,1) \forall t \) and \( i \)
- DG2: \( u_{it} = (I_T - \Omega_T)\epsilon_{it} \)
- DG3: \( u_{it} = \gamma u_{i,t-1} + \epsilon_{it} \)

\( \Omega_T \) corresponding to SAR(1) model, \( \theta = 0.8 \)
- \( f_{it} \sim N(0,1), \forall t, \gamma \sim U[0.05, 0.99] \)
- Selected Results

5.1 Empirical Illustration

Is the Current Account Stationary?
- annual data for 129 countries and max 33 years
- considered variable: current account to GDP ratio

5.1.1 Empirical Results

6. Conclusions
- panel unit root tests useful in many applications
- cross sectional dependence important feature of real-life examples
- this study proposes finite sample modifications of existing robust test
- wild bootstrap turns out to be efficient way of immunizing PURT’s against CS dependence